

Solution to Sample Exam

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2. Calculate the current through the resistors shown as rectangles and the batteries in the circuit below.

Top line: $R_1 = 10 \text{ Ohms}$ and $\epsilon_1 = 5 \text{ volts}$. The arrow represents current I_1 .

Middle line: $\epsilon_2 = 25 \text{ volts}$.

Bottom line: $R_3 = 20 \text{ Ohms}$ and $\epsilon_3 = 5 \text{ volts}$. The arrow represents current I_3 .

Consider current through top to bottom row I_1 , I_2 and I_3 respectively.

	Direction	ϵ_1	R_1	ϵ_2	R_3	ϵ_3
I_1 (Amp)	$\rightarrow$	2	2	XXXXXX	XXXXXX	XXXXXX
I_2 (Amp)	$\leftarrow$	XXXXXX	XXXXXX	3.5	XXXXXX	XXXXXX
I_3 (Amp)	$\rightarrow$	XXXXXX	XXXXXX	XXXXXX	1.5	1.5

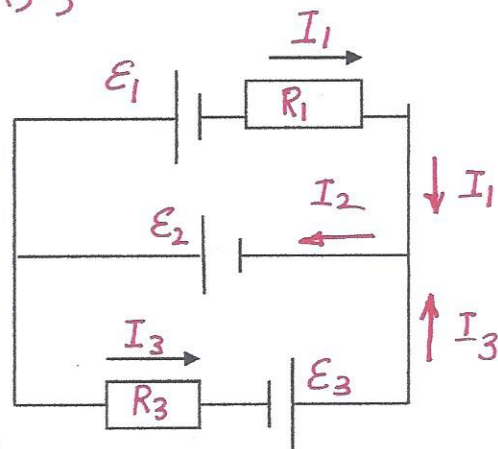
① Top loop (Clockwise) $-\epsilon_1 - R_1 I_1 + \epsilon_2 = 0$

② Bottom loop (CW) $-\epsilon_2 - \epsilon_3 + R_3 I_3 = 0$

① $-5 - 10 I_1 + 25 = 0$
 $I_1 = 2 \text{ Amp}$

② $-5 - 25 + 20 I_3 = 0$
 $I_3 = 1.5 \text{ Amp}$

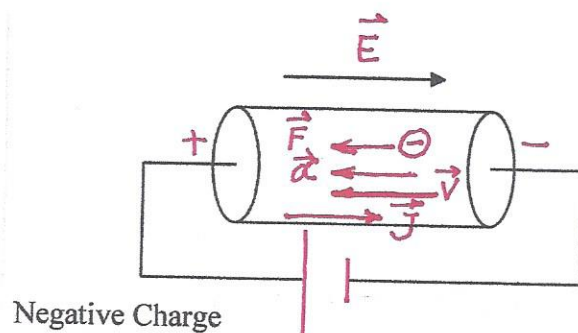
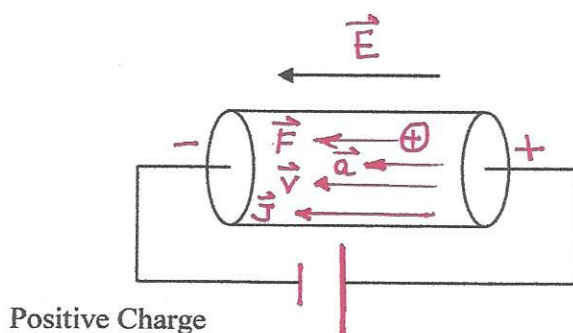
③ $-I_2 + I_1 + I_3 = 0$
 $I_2 = I_1 + I_3 = 2 + 1.5 = 3.5 \text{ Amp}$



1. The vector shown in each figure is the electric field.

a. Show the vectors: force, acceleration, velocity, and current density for the positive and negative charges shown below.

b. Show the polarity (positive and negative sides) of the battery that gives the electric field shown.



3. A 12-volt battery is connected in series with a resistor $R = 10,000$ Ohms, and a capacitor with capacitance $C = 20 \mu\text{F}$.

- a. At what time after the battery is connected does the charge is half maximum value?

$$q(t) = q_{\max} (1 - e^{-t/\tau}) \quad \tau = RC = 10 \times 20 \times 10^{-6} = 0.2 \text{ sec}$$

$$\frac{1}{2} q_{\max} = q_{\max} (1 - e^{-t/\tau})$$

$$e^{-t/\tau} = 0.5 \rightarrow -t/\tau = \ln 0.5, \quad t = -\tau \ln 0.5 = 0.69\tau$$

$$t = 0.69 \times 0.2 = 0.138 \text{ sec}$$

- b. At what time after the battery is connected does the current through resistor equals 1/3 of maximum value?

$$q(t) = CE(1 - e^{-t/RC})$$

$$i = \frac{dq}{dt} = \frac{E}{R} e^{-t/RC} = i_{\max} e^{-t/RC}$$

$$\frac{1}{3} i_{\max} = i_{\max} e^{-t/RC}, \quad e^{-t/RC} = \frac{1}{3}$$

$$-t/RC = \ln\left(\frac{1}{3}\right) = -1.098 \quad t = RC(1.098) = 0.2(1.098)$$

$$t \approx 0.22 \text{ sec.}$$

4. A proton and an alpha particle (2 protons plus 2 neutrons) both with same velocity v enter a region of magnetic field B where direction of B is perpendicular to direction of v .

- a. Calculate the ratio of radius of circular path of alpha particle to that of proton.

$$R_p = \frac{m_p v}{q_p B}, \quad R_\alpha = \frac{m_\alpha v}{q_\alpha B} = \frac{4m_p v}{2q_p B} = \frac{2m_p v}{q_p B}$$

$$R_\alpha / R_p = 2$$

- a. Calculate the ratio of period of the circular path of alpha particle to that of proton.

$$2\pi R = vT \quad T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

$$T_\alpha = \frac{(2\pi)(4m_p)}{2q_p B} = \frac{4\pi m_p}{q_p B}, \quad T_p = \frac{2\pi m_p}{q_p B}$$

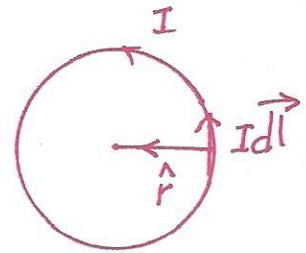
$$T_\alpha / T_p = 2$$

5. Calculate the expression of magnetic field B due to a circular coil with current I , radius R and number of turns N at the center of the coil as a function of I , N , and R .

$$d\vec{B} = \frac{\mu_0}{4\pi} I d\vec{l} \times \hat{r} / r^2 \quad dB = \frac{\mu_0}{4\pi} \frac{Idl}{R^2} \sin 90$$

$$B = \int \frac{\mu_0}{4\pi} \frac{Idl}{R^2} \quad \sin 90 = 1$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{R^2} \int dl = \frac{\mu_0 I}{4\pi} \frac{2\pi R}{R^2} = \frac{\mu_0 I}{2R}$$



7. Write T (true) or F (False) before each statement.

T The electric field inside a fully charge capacitor is at its maximum.

T When we connect two resistors in parallel the equivalent resistance is less than both resistances.

F When we connect two capacitors in parallel the equivalent capacitance is less than both capacitances.

T Positive charge is accelerated toward lower potential.

T Negative charge is accelerated toward higher potential.

T The larger the magnetic field the smaller the radius of circular motion of a moving

#6 has not been shown here since it is on chapter 24.